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CONTENT:

Relationship between temporal and spatial reflectogram scales in fiber OTDR systems

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Abstract

Correspondence between time instant of the reflectogram and position of the optical fiber sector at which the scattering occurred is analyzed for the longitudinally inhomogeneous optical fiber and the external perturbation of the general kind. Conditions on the rate of the refractive index and external perturbation dependence on the longitudinal coordinate are stated that imply explicit formulae for shifts of scattering segments of various position with respect to the external perturbation. The effects are numerically estimated in the cases of geometric elongation and heating of the fiber.

Keywords:

OTDR, fiber optic sensors, rayleigh scattering, distributed sensors, reflectogram scale

1. Introduction

Distributed measurement systems based on optical time domain reflectometry (OTDR) have found a wide application area in fiber optical communication line control and distributed fiber optic sensors development [1, 2]. Besides already existing for a long time devices aimed at measuring the fiber loss distribution, new classes of OTDR systems get their application areas. Distributed temperature sensors (based on Raman scattering), static tension sensors (based on Brillouin scattering) and coherent Rayleigh vibration sensors have been developed and successfully implemented [3, 4, 5, 6]. Since the light registration and digital signal processing methods have been significantly improved, their capabilities and efficiency are drastically enhanced.

In general OTDR measurements can be described as following: a light pulse of duration T_P is input through the fiber end. While propagating along the cable, light undergoes a Rayleigh scattering, which is partially trapped backwards, forming continuous backscattered wave. Its intensity $I(\tau)$ is recorded, resulting in a reflectogram signal. The argument τ is a time delay after the start of probe pulse propagation. In some cases backscattered wave suffers additional optical transformations, which, however, don't affect statistical properties of $I(\tau)$ dependence.

One of the most challenging tasks in OTDR measurements is attribution of a backscattered wave value $I(\tau_i)$ to a certain spatial interval, from which the considered part of the wave was reflected. Such a relation between spatial and temporal scales of reflectogram can be introduced in various manners, and in numerous papers, dealt with this topic [3, 4, 5], different expressions for $I(\tau_i)$ are proposed. The essence of the traditional approach can be explained as follows. Given the delay of the light wave as it propagates to the scattering sector and back, one can express the intensity of the backward wave $I(t)$, produced by the probe pulse in the following form

$$I(t) = \int_0^L I_0\left(t - \frac{x}{v_g}\right)dx, \tag{1}$$

where $v_g = c/n$ (c and n – light speed in vacuum and group refraction index of single mode fiber), L is the fiber cable length. Considering finite time limits of the probe pulse, the function $I_0(t)$ characterizing the scattering process is different from zero within the interval $[-T_P/2; T_P/2]$, therefore the integrand in (1) isn't equal to zero only within some specific spatial interval and for any specified value τ_i expr. (1) can be rewritten as

$$I(\tau_i) = \int_{z_{i1}}^{z_{i2}} I_0\left(\tau_i - \frac{x}{v_g}\right)dx,$$

$$z_i = (z_{i1}+z_{i2})/2 = \tau_i \cdot c/2n, \quad \Delta z_i = z_{i2} - z_{i1} = T_P \cdot c/2n. \tag{2}$$

In such a manner, with each moment τ_i, can be associated a scattering sector, located between z_{i1} and z_{i2} of length Δz. Such a consideration assumes the values v_g and n to be constant over the fiber length, which takes place in all known literature related to OTDR measurements.

However, during the real-world measurements, the value of refractive index can vary with respect to time and spatial coordinate. It’s worth while mentioning that deviations of n can be caused by target (measured) perturbations and outer condition changes. The question about the adequacy of expression (2) becomes even more actual in view of latter attempts of increasing measurement accuracy.

In the present work we propose a strict consideration of spatial and temporal scales relation with taking into account influences of refraction index deviations.

2. Generalized description of reflecting sector's center and bound coordinates in unperturbed fiber line

As the starting point for strict consideration of temporal and spatial reflectogram scales let us introduce an expression for the time delay of the light wave propagating along the fiber with a fixed distribution of the refractive index $n(z)$ from the beginning to a certain point with the coordinate z_0 and back

$$\tau = \frac{2}{c}\int_0^{z_0} n(x)dx. \tag{3}$$

Since the probe pulse has a certain finite length, at instant τ_i a superposition of partial waves, scattered by different fiber points situated inside the sector $[z_{i1}; z_{i2}]$, being excited by different parts of the probe pulse, are simultaneously received by photodetector. In this case sector bounds can be defined by the following expression

$$\tau_{i1,2} = \frac{2}{c}\int_0^{z_{i1,2}} n(x)dx, \tag{4}$$

where τ_{i1} and τ_{i2} are time delays between the backscattered wave registration moment τ_i and moments for fore- and back-fronts of the probe pulse to pass the fiber input end.

$$\tau_{i1} = \tau_i - T_P/2 \quad \text{and} \quad \tau_{i2} = \tau_i + T_P/2. \tag{5}$$

Let us express relations (4) and (5) in terms of light registration moment τ_i and center position of the corresponding scattering sector in order to clarify their analysis. The position of sector center is given by expression

$$\tau_i = \frac{2}{c}\int_0^{z_i} n(x)dx, \tag{6}$$

scattering sector bounds z_{i1} and z_{i2} are related to τ_i as follows:

$$\tau_i = \frac{2}{c}\int_0^{z_{i1}} n(x)dx + \frac{T_P}{2} = \frac{2}{c}\int_{z_i}^{z_{i2}} n(x)dx - \frac{T_P}{2}. \tag{7}$$

In such a manner, scattering sector bounds can be defined by expressions

$$z_{i1} = z_i + \Delta z_{i1};\ z_{i2} = z_i + \Delta z_{i2}, \tag{8}$$

where $\Delta z_{i1,i2} = z_{i1,2} - z_i$ are the bounds' displacements with respect to the sector center deduced from the expression

$$\frac{T_P}{2} = \pm\frac{2}{c}\int_{z_i}^{z_{i1,2}} n(x)dx. \tag{9}$$

The total length of the scattering sector, related to the *i*-th reflectogram point can be presented as

$$\Delta z(z_i) = z_{i2} - z_{i1} = \Delta z_{i2} - \Delta z_{i1}. \tag{10}$$

Thus relations given by (4) and (5) can be expressed in terms of probe pulse spatial coordinate and lengths z_i and Δz_{i1}, Δz_{i2} (defined by expressions (6), (8) or (9), respectively).

Let us consider an important practical case where the probe pulse (and scattering sector) is relatively short. Then the value of refractive index can be supposed constant within any considered scattering sector. In such a case we can see that $|\Delta z_{i1}|=|\Delta z_{i2}|$ and the total length of the scattering sector is given by expression

$$\Delta z(z_i) = \frac{T_P}{2} \cdot \frac{c}{n(z_i)} \quad . \tag{11}$$

This approximation is similar to widely used expression (2), however, in most cases *n* is supposed to be constant.

3. Perturbations of the fiber line under the test

Let the fiber line be perturbed with spatial distribution $V(z)$. In order to take into account how disturbances affect the position and length of the scattering sector, one needs to know how to convert the initial dependence $n(z)$ into a new distribution $n'(z)$ which describes the refractive index after the application of perturbation.

To consider such an effect a factor K_{Vn} is to be introduced, which defines a direct change of the fiber refractive index caused by this type of perturbation *V*

$$K_{Vn} = \left.\frac{dn}{dV}\right|_{V=0} . \tag{12}$$

This coefficient characterizes the effect of a certain physical influence. Let us consider such vivid and practical cases as mechanical strain and temperature deviations. Mechanical strain is characterized by fiber relative elongation ε, and in the case of distributed perturbation – by function $\varepsilon(z)$. These perturbations are typically of order 10^{-6} and their unit is microstrain. According to frequently used estimations, fiber elongation causes *n* decrement and $K_{\varepsilon n}$ is of value around – 0,3 ÷ – 0,6. Temperature perturbations can be defined by distribution of temperature deviation $\Delta T(z)$. Widespread estimations of its value are $K_{Vn} \sim (0{,}6 \div 2) \cdot 10^{-5}$ [K^{-1}].

In order to produce a strict analysis of the spatial dependence of $n(z)$ one must also take into account the effect of deformation of the fiber points' initial coordinates caused by the perturbation. For this purpose a coefficient $K_{Vz} = d\varepsilon/dV|_{V=0}$ must be taken into consideration. Fiber elongation is related to mechanical strain by factor $K_{\varepsilon z}$=1 and to temperature deviations by factor $K_{\Delta Tz} \sim 10^{-7}$ [K^{-1}].

In general, $n(z)$ deviations are caused by two effects: direct change stemming from the perturbation, accounted by K_{vn} coefficient; and indirect one, related to spatial scale transformation, described by K_{vz}. On one hand, second mechanism has complex and bulky description and on the other hand, in practical cases for mechanical strain and temperature deviations, indirect mechanism can be neglected comparing to the direct one.

Another important assumption concerns the maximal rate of perturbation alterations. It must be of such value, that perturbation distribution $V(z)$ be constant while the light propagates along the fiber line and backwards. Alternative case isn't considered since its bulkiness. Taking into account latter assumption, new distribution $n'(z)$, caused by fiber perturbations, can be expressed via the initial one with the following simple expression:

$$n'(z) = n(z) + K_{Vn}\cdot V(z) \qquad (13)$$

Let us mention a simpleand clear case of uniform perturbation on some spatial interval $[z_b; z_e]$, inside which the perturbation is constant $V(z)=V$. In this case an expression for $n'(z)$ can be expressed in the following form:

$$\begin{cases} n'(z) = n(z), & \text{if } z \notin [z_b; z_e], \\ n'(z) = n(z) + K_{Vn}\cdot V, & \text{if } z \in [z_b; z_e]. \end{cases} \qquad (14)$$

On the basis of expressions (13) or (14), defining the change of refraction index spatial distribution, one can consider the change of the main parameters of the scattering sector, corresponding to i-th reflectogram point – position shift and length variation.

4. Shift of the scattering sector centre

Let's first of all consider a position shift of the scattering sector (sector centre position) under external perturbation $V(z)$. If a perturbed sector of the fiber $[z_b; z_e]$ is located before the scattering sector center, i.e. $z'_i > z_e$, then, so far as z_i and z'_i correspond to the same instant τ_i of the scattering wave arrival, their interdependence according to the basic relationship (6) is described with the expression

$$\tau_i = \frac{2}{c}\int_0^{z_i} n(x)dx = \frac{2}{c}\int_0^{z'_i} n'(x)dx \qquad (15)$$

Rewriting the second integral with taking into account (13) one obtains an equation

$$\int_0^{z_i} n(x)dx = \int_0^{z'_i} n(x)dx + \int_{z_b}^{z_e} K_{Vn}\cdot V(x)dx \qquad (16)$$

Thus for determining z'_i in general case it's necessary to solve the transcendent equation (16), a quantity to be found appearing to be a limit of integration.

An explicit relationship between z_i and z'_i can be obtained in a case where an assumption $n(z_i) \approx n(z'_i)$ could be done. This condition can be met in practice by, firstly, relative slow spatial variation of n, and, secondly, small shift magnitude of z'_i with respect to z_i. If this condition is obeyed then (16) can be transformed to the relationship

$$z'_i = z_i - \frac{\int_{z_b}^{z_e} K_{Vn} \cdot V(x)dx}{n(z_i)} \tag{17}$$

Notice that (17) is derived in the case of original condition which isn't suitable for practical use. If an original dependence $n(z)$ is known and one needs to estimate a possible shift of a sector with the center z_i on account of influence on the fiber, validity of the condition $z'_i > z_e$ isn't apparent until such an estimate is done. From this viewpoint, taking into account (17), the condition $z'_i > z_e$ may be rewritten using the original value z_i as

$$z_i > z_e + \frac{\int_{z_b}^{z_e} K_{Vng}\, V(x)\, dx}{n_g(z_i)} \tag{18}$$

Let's suppose a situation where the shifted center z'_i of the scattering sector is situated within the perturbed segment $[z_b; z_e]$ of the fiber, i.e. the condition $z_b < z'_i < z_e$ holds. In this case from basic relationships (6) and (13) a relation between z_i and z'_i is obtained as

$$\int_0^{z_i} n(x)dx = \int_0^{z'_i} n(x)dx + \int_{z_b}^{z'_i} K_{Vn} \cdot V(x)dx \tag{19}$$

If it's admissible to assume that the approximation $n(z_i) \approx n(z'_i)$ is met with a sufficient accuracy then more concrete formulae can be presented. The condition $z_b < z'_i < z_e$, with the formulated condition (18) being taken into account, can be written down via z_i as follows

$$z_b < z_i < z_e + \frac{\int_{z_b}^{z_e} K_{Vn} \cdot V(x)dx}{n(z_i)}, \tag{20}$$

and estimate of the shift z'_i is given with the relationship

$$z'_i = z_i - \frac{\int_{z_b}^{z'_i} K_{Vn} \cdot V(x)dx}{n(z_i)}, \tag{21}$$

which contrary to (17) turns out a transcendent equation requiring numeric solution.

And at last in case where the perturbed segment of the light guide is situated behind the shifted scattering sector under consideration, then the perturbation is obvious not to affect the sector's center position, since in propagation to the point z'_i and reverse, the light doesn't pass the perturbed segment. This case is characterized with the condition $z'_i < z_b$, with this time it corresponding to as simple condition $z_i < z_b$.

Summarizing, if the approximate equality $n(z_i) \approx n(z'_i)$ may be supposed, the laws of shift of the scattering sector center can be presented as follows

$$\begin{aligned}
z_i' &= z_i - \frac{\int_{z_b}^{z_e} K_{Vn}\cdot V(x)dx}{n(z_i)}, \quad \text{if } z_i > z_e + \frac{\int_{z_b}^{z_e} K_{Vn}\cdot V(x)dx}{n(z_i)};\\
z_i' &= z_i - \frac{\int_{z_b}^{z_i'} K_{Vn}\cdot V(x)dx}{n(z_i)}, \quad \text{if } z_b < z_i < z_e + \frac{\int_{z_b}^{z_e} K_{Vn}\cdot V(x)dx}{n(z_i)};\\
z_i' &= z_i, \quad \text{if } z_i < z_b.
\end{aligned} \tag{22}$$

On one hand, expressions and conditions in (22) turn out simpler and more definite than more general relationships (16), (19) and conditions $z' \notin [z_b; z_e]$ or $z' \in [z_b; z_e]$ that can be verified actually on solving the equations. But they remain relatively cumbersome since they contain integrals, and equation in the second line remains transcendent. Therefore for a visual analysis of regularities and numerical estimates it's expedient to regard an example of a relatively simple situation where the segment $[z_b; z_e]$ is exposed to a uniform influence V. In this case the integrals and the transcendent relationship can be excluded. Putting $V(x)$=const in (22) one can readily obtain the required relationships

$$\begin{aligned}
z_i' &= z_i - \frac{K_{Vn}\cdot V(z_e - z_b)}{n(z_i)}, \quad \text{if } z_i > z_e + K_{Vn}\cdot V(z_e - z_b)/n(z_i);\\
z_i' &= \frac{z_i\cdot n(z_i) + K_{Vn}\cdot V\cdot z_b}{n(z_i) + K_{Vn}\cdot V}, \quad \text{if } z_b < z_i < z_e + K_{Vn}\cdot V(z_e - z_b)/n(z_i);\\
z_i' &= z_i, \quad \text{if } z_i < z_b.
\end{aligned} \tag{23}$$

5. Bound shift and change in the scattering sector length

So far as expression (4) for $z_{i1,2}$ is analogous by structure and physical meaning to expression (6) for z_i then the results stated for the shift of the scattering sector center may be applied to the analysis of shift of the sector bounds. Therefore an interrelation between $z'_{i1,2}$ and $z_{i1,2}$ can be written down as

$$\begin{aligned}
&\int_0^{z_{i1,2}} n_g(x)dx = \int_0^{z_{i1,2}'} n_g(x)dx + \int_{z_b}^{z_e} K_{Vng}\cdot V(x)dx, \quad \text{if } z_{i1,2}' > z_e;\\
&\int_0^{z_{i1,2}} n_g(x)dx = \int_0^{z_{i1,2}'} n_g(x)dx + \int_{z_b}^{z_e} K_{Vng}\cdot V(x)dx, \quad \text{if } z_b < z_{i1,2}' < z_e;\\
&z_{i1,2}' = z_{i1,2}, \quad \text{if } z_{i1,2}' < z_b.
\end{aligned} \tag{24}$$

Like in consideration of the scattering sector center shift, presented above, in the case of approximate equality $n(z_{i1,2}) \approx n(z'_{i1,2})$ to $z'_{i1,2}$ could be applied quite the same expressions that have been obtained for z'_i (i.e. formula (22) or (23) for uniform perturbation), with z'_i, z_i and $n(z_i)$ being replaced with $z'_{i1,2}$, $z_{i1,2}$ and $n(z_{i1,2})$ respectively. Hence one can deduce required expressions for the scattering sector length $\Delta z'_i = z'_{i2} - z'_{i1}$, changed due to influence on the fiber.

Nevertheless, so far as the extent of the scattering sector depends on two bounds, their position with respect to the perturbed segment bounds admits six variants which are presented in fig.1:

1) sector before the perturbed segment, 2) end bound within the perturbed segment, 3) the perturbed segment within the scattering sector, 4) sector's beginning within the perturbed segment, 5) sector within the perturbed segment, 6) sector behind the perturbed segment.

Let's consider these variants in more details:

1) Sector is situated before the perturbed segment. This case corresponds to the condition $z_{i2} < z_b$. In this case sector's extent doesn't change and is described with expressions (8)–(10) or (11) with the original distribution $n(z)$.

2), 3) The situation where sector's beginning is situated before the beginning of the perturbed segment, and sector's end – behind the beginning of the perturbed segment, this corresponds to the conditions $z_{i1} < z_b$ and $z_{i2} > z_b$. In this case the shift z_{i2} should be determined proceeding from (17) or (21), replacing $n(z_i)$ with $n(z_{i2})$. So far as in these cases $z_{i1} = z'_{i1}$ then $\Delta z'_i = \Delta z'_i + z'_{i2} - z_{i12}$.

4), 5) In these cases (sector's beginning within the perturbed segment or the whole sector within the perturbed segment) it's necessary to determine shift of both sector bounds in accordance to (17) or (21) (replacing $n(z_i)$ with $n(z_{i1,2})$) and calculate sector's extent as $\Delta z'_i = \Delta z'_{i2} - z'_{i1}$. In this case the conditions $z_b < z'_{i1} < z_e$ and $z'_{i2} > z_e$ are met. For the original position of sector's beginning (prior to perturbation) the conditions take the form

$$z_\mathrm{b} < z_{i1} < z_e + \frac{1}{n(z_{i1})}\int_{z_b}^{z_e} K_{Vn} \cdot V(x)dx, \quad \text{if } V(z),$$
$$z_\mathrm{b} < z_{i1} < z_e + K_{Vn} \cdot V \cdot (z_e - z_b)/n(z_{i1}), \quad \text{if } V = \text{const}. \tag{25}$$

6) A relatively simple case where sector is situated behind the perturbed segment, i.e. $z'_{i1} > z_e$. As one can conclude from (22), this condition corresponds to the relationship

$$z_{i1} > z_e + \frac{\int_{z_b}^{z_e} K_{Vng} \cdot V(x)dx}{n_g(z_{i1})}, \tag{26}$$

and expression for sector's length variation can be written down as

$$\Delta z_i' = z_{i2}' - z_{i1}' = \Delta z_i + \int_{z_b}^{z_e} K_{Vn} \cdot V(x)\, dx \cdot \frac{n(z_{i2}) - n(z_{i1})}{n(z_{i2})n(z_{i1})}. \quad (27)$$

Results obtained for calculation of the scattering sector length increment caused by influence on the light guide, and conditions under which one or another expression should be used for such a calculation are aggregated in Table 1.

And at last, relationships can be noted which are applicable in the situation where the refractive index *n* is admissible to be assumed constant over the whole sector extent. In this case the relationships become much simpler.

Length of the sector located behind the perturbed segment is unchanged.

For the sector within which the perturbed segment is situated the following relationships are valid

$$z_{i1} < z_b,\ z_{i2} > z_e + \frac{K_{Vn}V(z_e - z_b)}{n(z_i)};\ \ \Delta z_i' = \Delta z_i - \frac{K_{Vn}V(z_e - z_b)}{n(z_i)}. \quad (28)$$

For the sector within the perturbed segment the expressions are obtained

$$z_{i1} > z_b,\ z_{i2} < z_e + \frac{K_{Vn}V(z_e - z_b)}{n(z_i)};\ \ \Delta z_i' = \frac{\Delta z_i}{1 + K_{Vn}V/n(z_i)}. \quad (29)$$

These results are aggregated in Table 2.

In order to illustrate a possible influence of perturbations one can carry out some coarse estimates for the simplest and visual cases. Let the probe pulse have duration 100 ns which corresponds to the scattering sector length about 10 m. Suppose the fiber line segment of 1 km length is heated by 20^0C and put n=1.5 and $K_{\Delta Tn}$= 10^{-5} $1/^0$C, then by means of expression (23) the scattering sector center shift can be readily evaluated to have a magnitude about 13 cm. If a scattering sector is situated within the perturbed segment then the change in its extent comprises about 1.3 mm. How significant are these shifts and scattering sector parameter variations in cases more complicated for accounting, regarded above, depends on a concrete type of OTDR device, type of the measured quantity and principle of its determination, accuracy characteristics of the system.

In conclusion it should be noted that for the sake of completeness of the research carried out, one has to regard requirements of validity of several approximations that have been used, in order to pass from general integral equalities to explicit expressions for the estimates (e.g. neglecting geometric elongation, holding $n(z_i) \approx n(z'_i)$). For the correction related to these approximations to be much less than the very variations of the scattering pulse parameters, certain inequalities must hold including n, dn/dz, K_{vn}, K_{vz} etc. Investigation of these tasks might appear a subject of a separate paper.

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Figure captions:

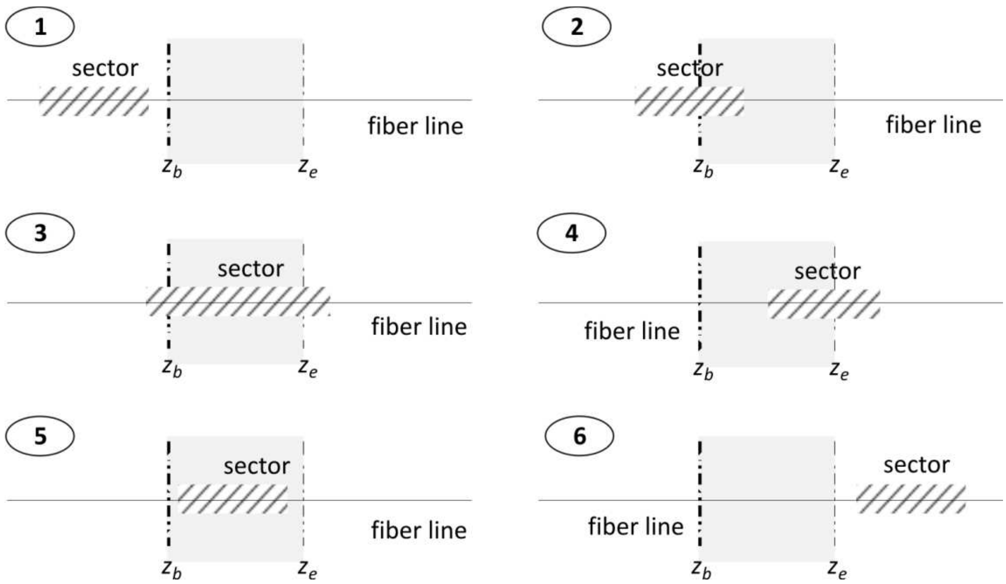

Fig. 1. Variants of position of the scattering sector bounds with respect to the perturbed segment.

Tables:

Table 1. Estimative expressions for Δz_i (if $n(z_{i1,2}) \approx n(z'_{i1,2})$)

	Condition for z_{i1}	Condition for z_{i2}	Formula
1)	–	$z_{i2} < z_b$	$\Delta z'_i = \Delta z_i$
2) 3)	$z_{i1} < z_b$,	$z_{i2} > z_b$	Calculation of z'_{i2} $\Delta z'_i = \Delta z_i + (z'_{i2} - z_{i2})$ 2) – (17); 3) – (21)
4) 5)	$z_b < z_{i1} < z_e + \frac{\int_{z_b}^{z_e} K_{Vn} V(x) dx}{n(z_{i1})}$ $z_b < z_{i1} < z_e + \frac{K_{Vn} V(z_e - z_b)}{n(z_{i1})}$	–	Calculation of z'_{i1} and z'_{i2}, $\Delta z'_i = z'_{i2} - z'_{i1}$ 4) – (17) 5) – (21)
6)	$z_{i1} > z_e + \frac{\int_{z_b}^{z_e} K_{Vn} V(x) dx}{n(z_{i1})}$ $z_{i1} > z_e + \frac{K_{Vn} V(z_e - z_b)}{n(z_{i1})}$	–	$\Delta z'_i = \Delta z_i + \frac{n(z_{i2}) - n(z_{i1})}{n(z_{i2}) n(z_{i1})} \int_{z_b}^{z_e} K_{Vn} V(x) dx$ $\Delta z'_i = \Delta z_i + \frac{n(z_{i2}) - n(z_{i1})}{n(z_{i2}) n(z_{i1})} K_{Vn} V(z_e -$

Table 2. Estimative expressions for Δz_i (if $n(z_{i1}) \approx n(z_{i2}) \approx n(z_i)$)

	Condition for z_{i1}	Condition for z_{i2}	Formula
3)	$z_{i1} < z_b$,	$z_{i2} > z_e + \frac{K_{Vn} V(z_e - z_b)}{n(z_i)}$	$\Delta z'_i = \Delta z_i - \frac{K_{Vn} V(z_e - z_b)}{n(z_i)}$
5)	$z_{i1} > z_b$,	$z_{i2} < z_e + \frac{K_{Vn} V(z_e - z_b)}{n(z_i)}$	$\Delta z'_i = \frac{\Delta z_i}{1 + K_{Vn} V / n(z_i)}$
6)	$z_{i1} > z_e + \frac{K_{Vn} V(z_e - z_b)}{n(z_i)}$	–	$\Delta z'_i = \Delta z_i$